

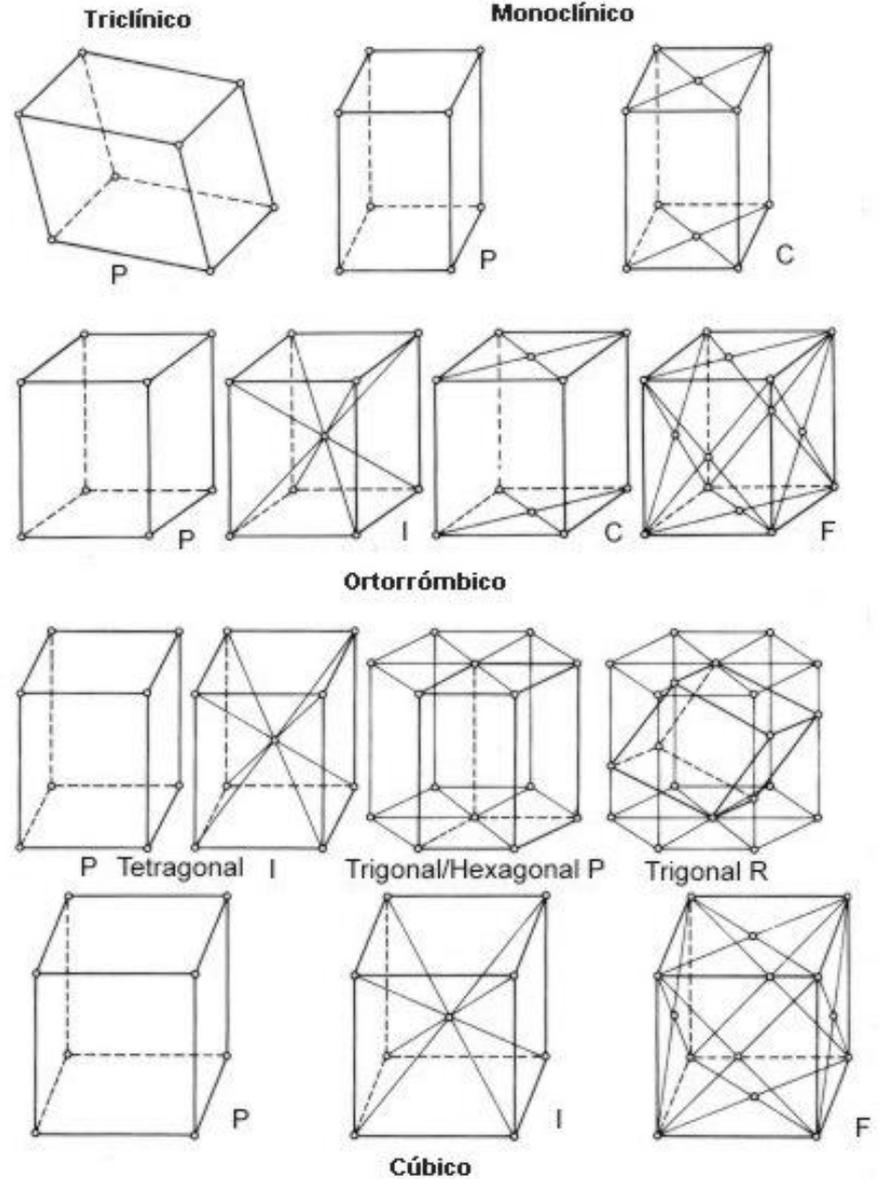
TEMA 4

Cristalografía geométrica II

LAS 14 REDES DE BRAVAIS



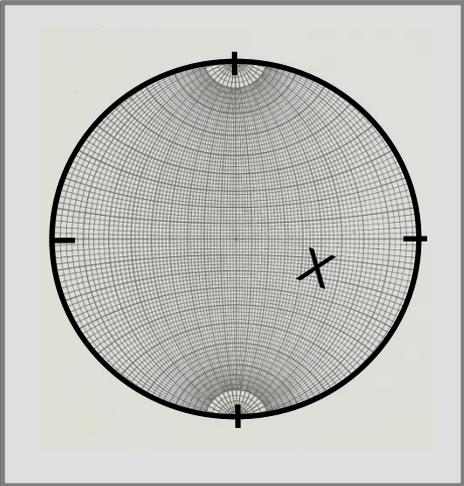
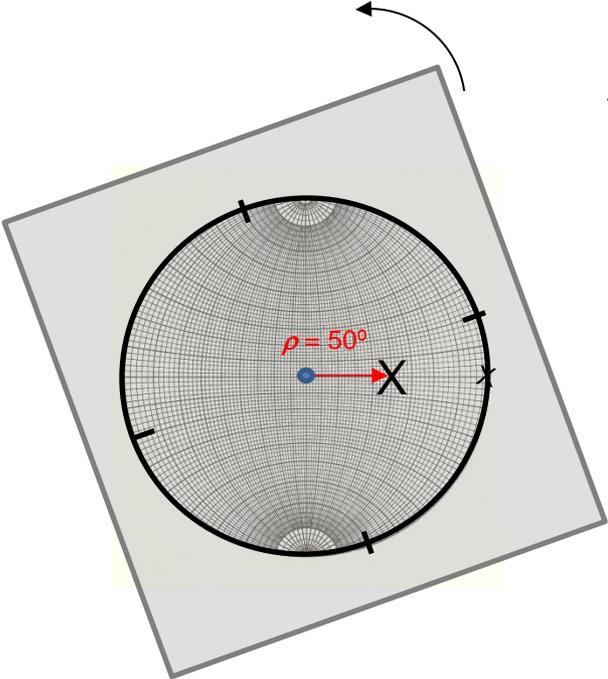
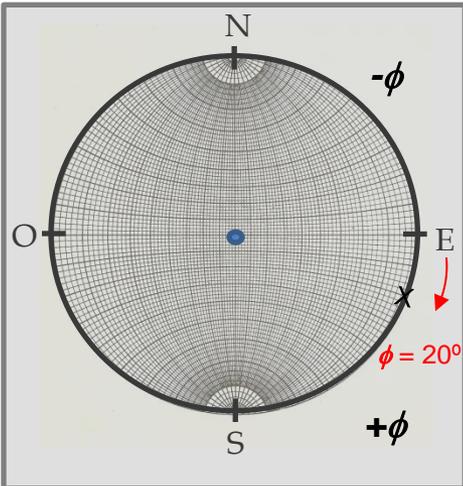
Auguste Bravais
(1811-1863)



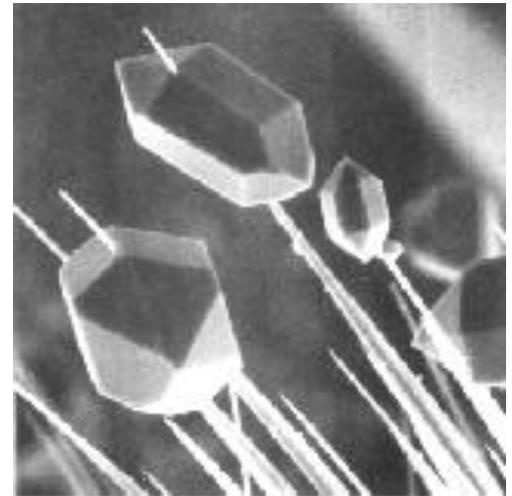
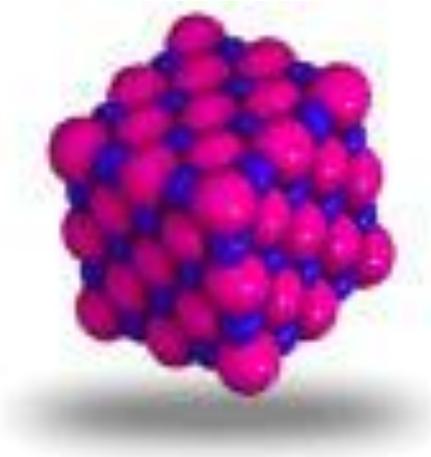
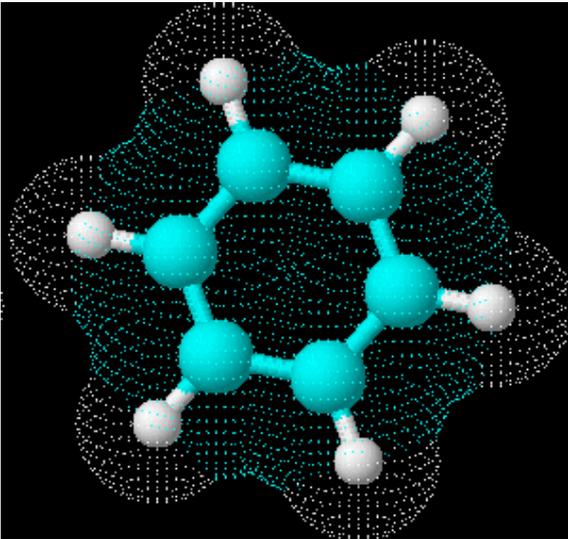
Empleo de la falsilla de Wulff

Longitud
Colatitud

$$\phi = 20^\circ$$
$$\rho = 50^\circ$$



Simetría de las 32 clases cristalinas



<http://minerals.gps.caltech.edu>





Johann F. C. Hessel
(1796-1872)

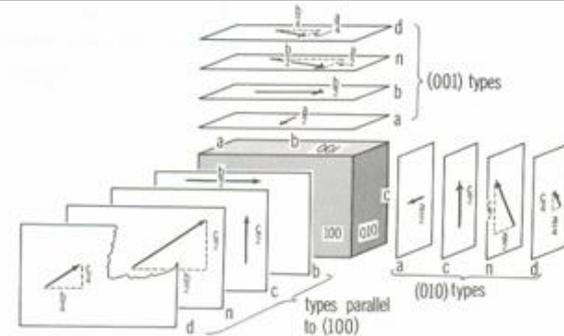
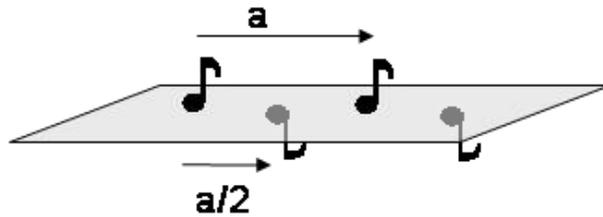
Los 32 grupos de simetría puntual tridimensional

Trigonal	Hexagonal	Cubic	Triclinic	Monoclinic	Tetragonal
3	6	23	1	2	4
$\bar{3}$	$\bar{6}$		$\bar{1}$	$m (= 2)$	$\bar{4}$
	6/m	$m\bar{3}$		2/m	4/m
				Orthorhombic	
32	622	432		222	422
3m	6mm			$mm\bar{2}$	4mm
	$\bar{6}m2$	43m			$\bar{4}2m$
$\bar{3}m$	6/mmm	$m\bar{3}m$		mmm	4/mmm

La derivación de los 157 grupos espaciales restantes precisa de la introducción de dos tipos de elementos de simetría nuevos: los planos de deslizamiento y los ejes helicoidales.

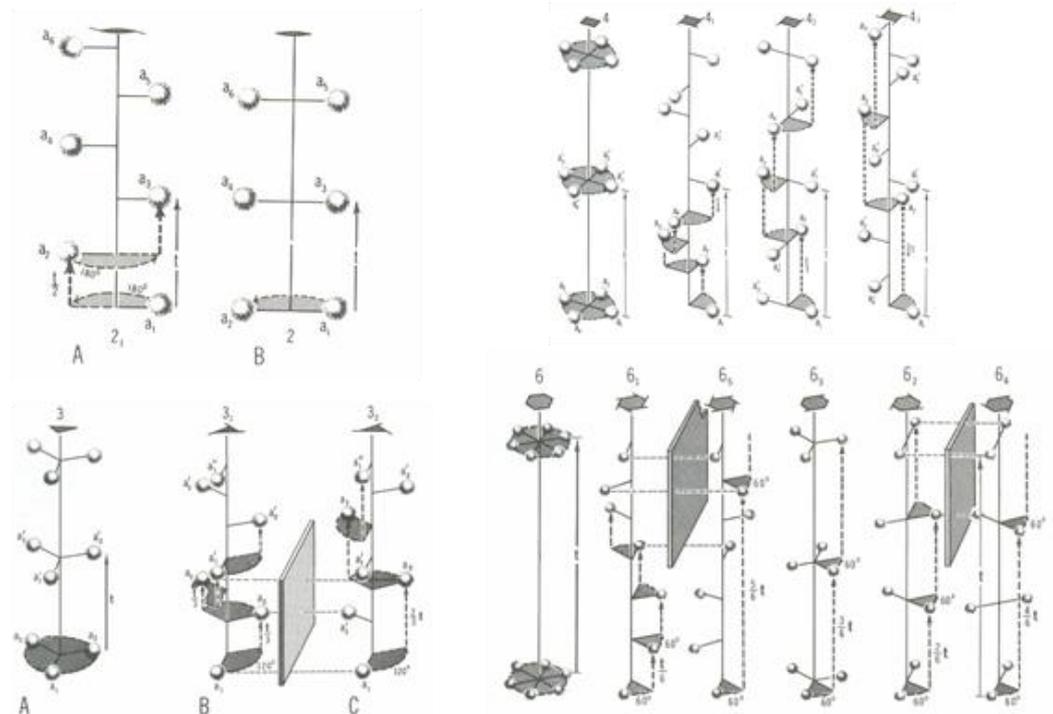
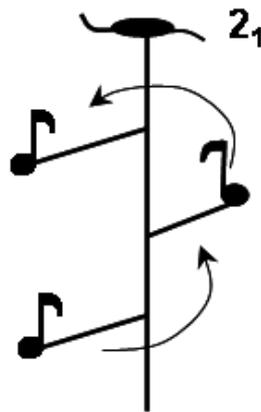
Planos de deslizamiento:

Se generan al combinar una reflexión con una translación

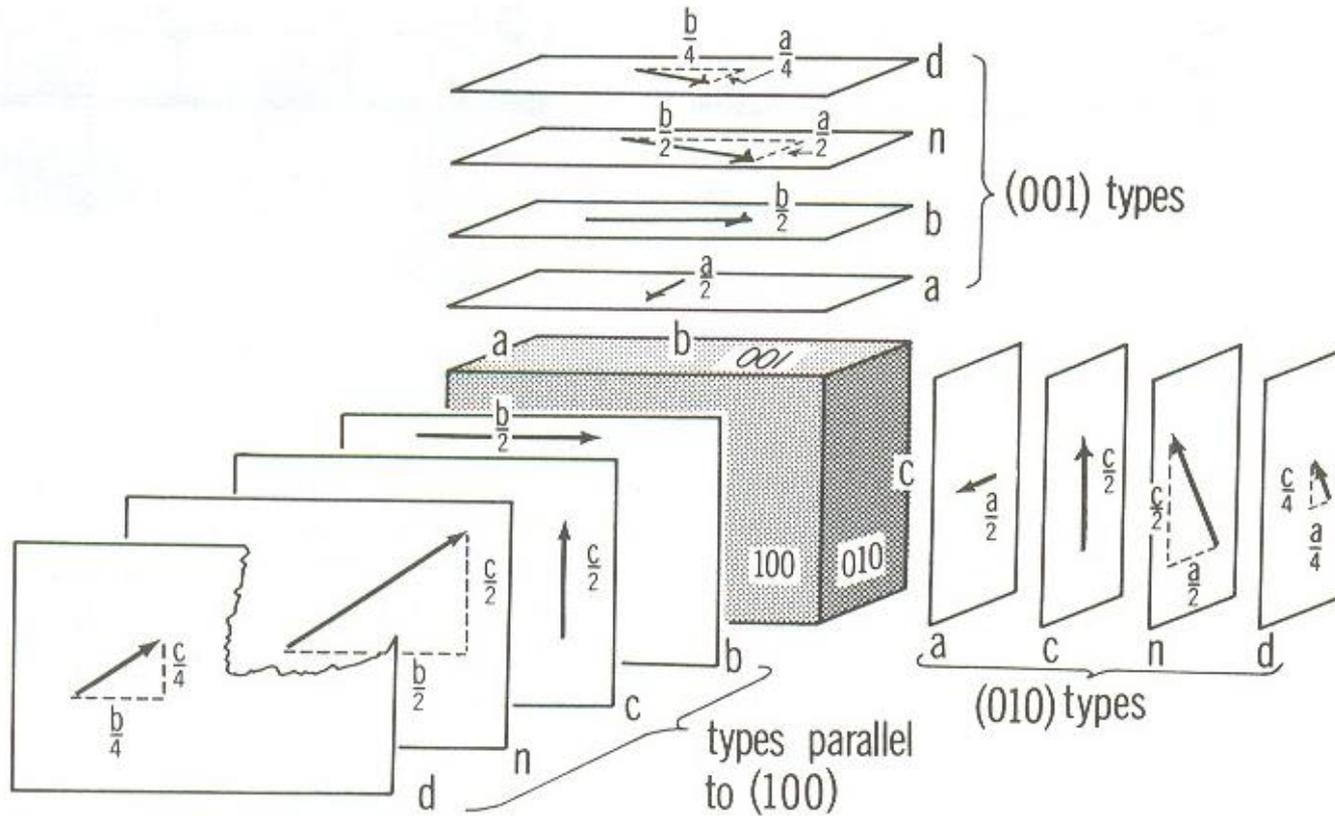


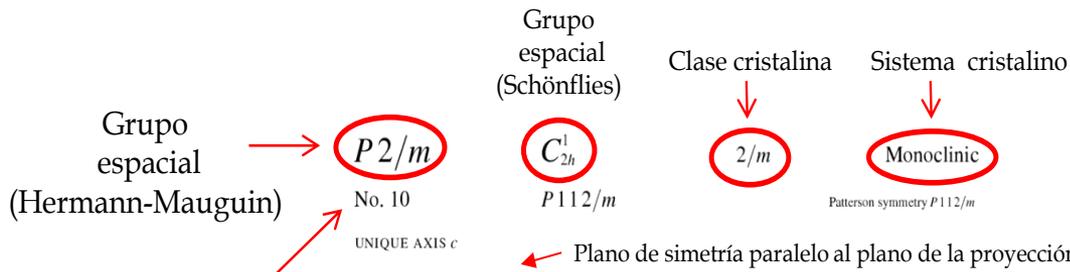
Ejes helicoidales:

En tres dimensiones existe también la posibilidad de combinar la rotación con la translación a lo largo del eje de simetría



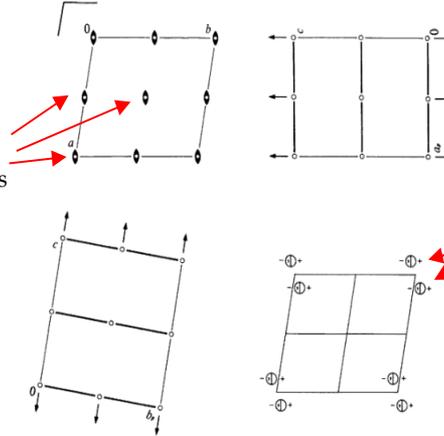
Planos de deslizamiento





Número del grupo espacial

Ejes binarios perpendiculares al plano de la proyección



Motivos equivalentes por simetría

Origin at centre ($2/m$)
 Generators selected (1); $r(1,0,0)$; $r(0,1,0)$; $r(0,0,1)$; (2); (3)

Positions	Multiplicity, Wyckoff letter, Site symmetry	(1) x, y, z	(2) \bar{x}, \bar{y}, z	(3) \bar{x}, y, \bar{z}	(4) x, \bar{y}, \bar{z}
4	o 1	x, y, z	\bar{x}, \bar{y}, z	\bar{x}, y, \bar{z}	x, \bar{y}, \bar{z}
2	n m	$x, y, \frac{1}{2}$	$\bar{x}, \bar{y}, \frac{1}{2}$		
2	m m	$x, y, 0$	$\bar{x}, \bar{y}, 0$		
2	l 2	$\frac{1}{2}, \frac{1}{2}, z$	$\frac{1}{2}, \frac{1}{2}, \bar{z}$		
2	k 2	$\frac{1}{2}, 0, z$	$\frac{1}{2}, 0, \bar{z}$		
2	j 2	$0, \frac{1}{2}, z$	$0, \frac{1}{2}, \bar{z}$		
2	i 2	$0, 0, z$	$0, 0, \bar{z}$		
1	h $2/m$	$\frac{1}{2}, \frac{1}{2}, \frac{1}{2}$			
1	g $2/m$	$\frac{1}{2}, \frac{1}{2}, 0$			
1	f $2/m$	$\frac{1}{2}, 0, \frac{1}{2}$			
1	e $2/m$	$0, \frac{1}{2}, \frac{1}{2}$			
1	d $2/m$	$0, \frac{1}{2}, 0$			
1	c $2/m$	$\frac{1}{2}, 0, 0$			
1	b $2/m$	$0, 0, \frac{1}{2}$			
1	a $2/m$	$0, 0, 0$			

Reflection conditions
 General:
 no conditions
 Special: no extra conditions

Condiciones de difracción

Coordenadas de las posiciones equivalentes por simetría (Posiciones de Wyckoff)

Symmetry of special projections
 Along [001] $p2$
 $a' = a$ $b' = b$
 Origin at $0, 0, z$

Along [100] $p2mm$
 $a' = b$ $b' = c$
 Origin at $x, 0, 0$

Along [010] $p2mm$
 $a' = c$ $b' = a$
 Origin at $0, y, 0$

Tablas Internacionales de la Cristalografía

$C m m 2$

No. 35

C_{2v}^{11}

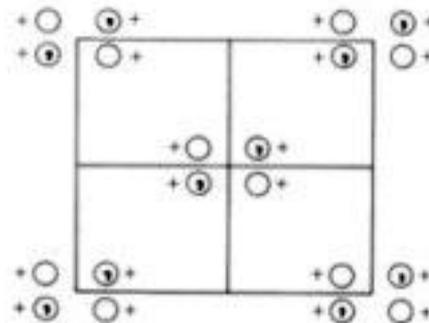
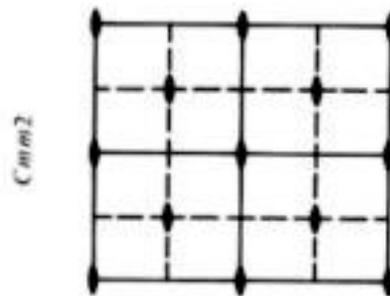
$C m m 2$

$m m 2$

Orthorhombic

Patterson symmetry $C m m m$

$C m m 2$



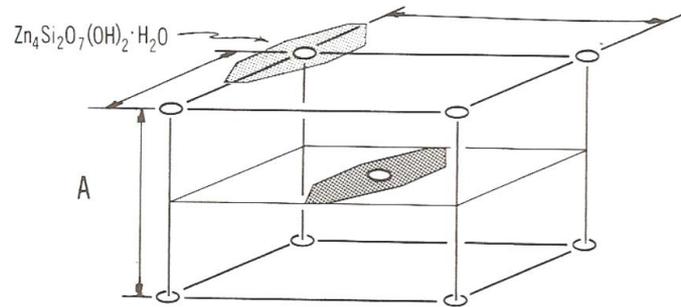
Positions

Multiplicity,
Wyckoff letter,
Site symmetry

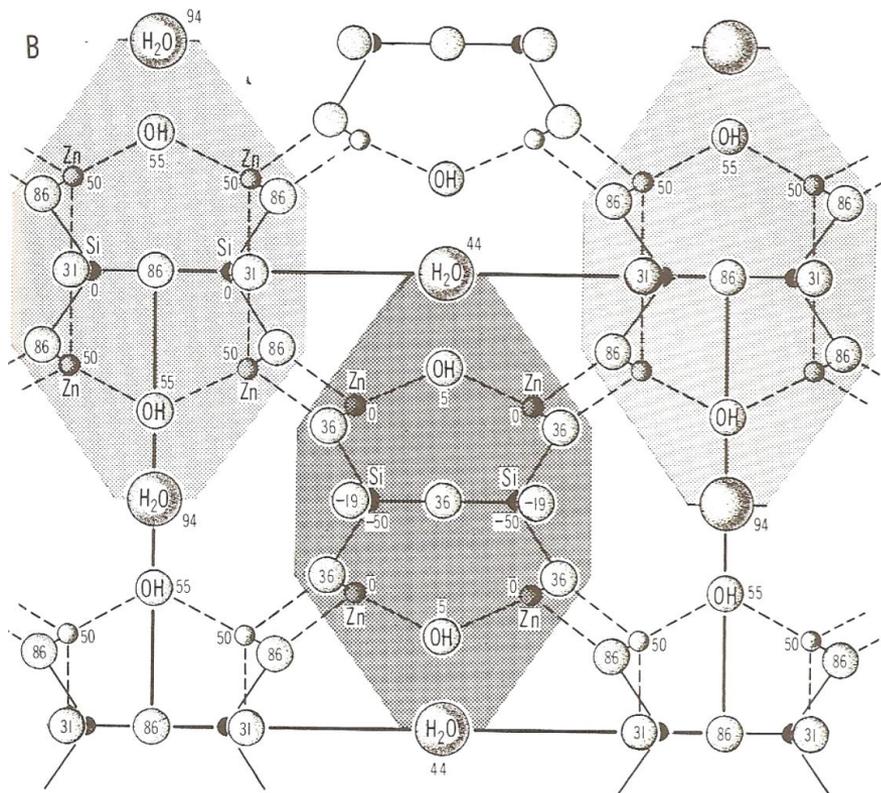
Coordinates

8 *f* 1 (1) x, y, z (2) \bar{x}, \bar{y}, z (3) x, \bar{y}, z (4) \bar{x}, y, z

$(0,0,0)+$ $(\frac{1}{2}, \frac{1}{2}, 0)+$



Hemimorfită
 Grupo espacial: $I mm2$



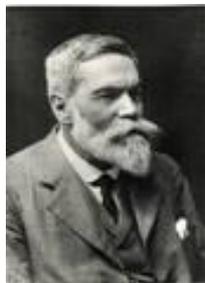
Grupos Espaciales



**Arthur Moritz Schönflies
(1853-1928)**



**Evgraf Stepanovich Fedorov
(1853-1919)**



**William Barlow
(1845-1934)**

	Triclinico	C222	Fdd2	Cmma	422	P4b2	Trigonal	Hexagonal	Cúbico	P43n
	1	C222 2 ₁ 2 ₁ 2	Fdd2 dd2 ₁	Cmma bab	422 P422	P4b2 P4n2	3	6	23	F43c
	P1	F222 2 ₁ 2 ₁ 2 ₁	Imm2 nn2 ₁	Ccca nnb	P4 ₂ 2 P4 ₁ 22	P4n2 I4m2	P3	P6	P23	I43d
	1̄	I222 2 ₁ 2 ₁ 2 ₁	Iba2 cc2 ₁	Fmmm bca cab nnn	P4 ₁ 22 P4 ₁ 2 ₁ 2	I4c2 I42m	P3 ₁ P3 ₂	P6 ₁ P6 _s	F23	m3m Pm3m
	P1̄	I2 ₁ 2 ₁ 2 ₁ 222	Ima2 nc2 ₁	Fddd	P4 ₂ 22 P4 ₂ 2 ₁ 2	I42d	R3 3̄ P3	P6 ₂ P6 _i P6 _s	I23	Pn3n Pm3n
	Monoclinico	2	mm2	mmm	P4 ₂ 22	4/mmm	R3̄	6̄	I2 ₁ 3	Pn3m
	P2	Pmm2	Pmm2	Immm nnn	P4 ₂ 22	P4/mmm	32	P8	m3	Pn3m
	P2 ₁	Pmc2 ₁	Pnnn	Ibam ccn	P4 ₂ 2 ₁ 2	P4/mcc	P312	6/m	Pm3	Fm3m
	C2	Pcc2	Pccm	Ibca cab	I422	P4/mcc	P321	P6/m	Pn3	Fm3c
	m	Pma2	Pban	Imma nnb	I4 ₁ 22	P4/nbm	P3 ₁ 12	P6 ₂ /m	Fm3	Fd3m
	Pm	Pca2 ₁	Pmma		4mm	P4/nnc	P3 ₂ 1	P622	Fd3	Fd3c
	Pc	Pnc2	Pnna		P4mm	P4/nbc	P3 ₁ 12	P6 ₁ 22	Im3	Im3m
	Cm	Pmn2 ₁	Pmna	Tetragonal	P4bm	P4/nbc	P3 ₂ 1	P6 ₂ 22	Pa3	Ia3d
	Cc	Pba2	Pcca	4	P4 ₂ cm	P4/nmm	R32	P6 ₂ 22		
	2/m	Pna2 ₁	Pbam	P4	P4 ₂ cm	P4/nmm	3m	P6 ₂ 22	Ia3	
	P2/m	Pnn2	Pccn	P4 ₁	P4cc	P4/ncc	P3m1	P6 ₂ 22	432	
	P2 ₁ /m	Cmm2	Pbcm	P4 ₂	P4nc	P4 ₂ /mmc	P3c1	P6cc	P432	
	C2/m	ba2	Pnmm	P4 ₃	P4 ₂ nc	P4 ₂ /mcm	P31c	P6 ₂ cm	P4 ₂ 32	
	P2/c	Cmc2 ₁	Pnmm	I4	P4 ₂ bc	P4 ₂ /nbc	R3m	P6 ₂ mc	F432	
	P2 ₁ /c	bn2 ₁	Pmnn	I4 ₁	I4mm	P4 ₂ /nbc	R3c	P6 ₂ m2	F432	
	C2/c	Ccc2	Pmnn	I4 ₁	I4cm	P4 ₂ /nbc	P31m	P6 ₂ m2	F432	
		nn2	Pbcn	4̄	I4 ₁ md	P4 ₂ /nbc	P31c	P6 ₂ m2	I432	
		Amn2	Pbca	P4̄	I4 ₁ cd	P4 ₂ /nbc	P3m1	P6 ₂ m2	I432	
		nc2 ₁	Pnma	I4̄	I4 ₁ cd	P4 ₂ /nbc	P3m1	P6 ₂ m2	I432	
	222	Abm2	Pnma	4/m	4̄2m	P4 ₂ /nbc	P3c1	6/mmm	P4 ₂ 32	
	P222	cc2 ₁	Cmcm bnn	P4/m	P42m	P4 ₂ /nbc	R3m	P6/mmm	P4 ₂ 32	
	P222 ₁	Ama2	Cmca bnb	P4 ₂ /n1	P42c	P4 ₂ /nbc	R3c	P6/mcc	P4 ₂ 32	
	P2 ₁ 2 ₁ 2	ba2	Cmmm ban	P4/n	P42 ₁ m	P4 ₂ /nbc		P6 ₂ /mcc	I4 ₁ 32	
	P2 ₁ 2 ₁ 2 ₁	cn2 ₁	Cmmm ban	P4 ₂ /n	P42 ₁ c	P4 ₂ /nbc		P6 ₂ /mcc	43m	
	C222 ₁	Fmm2	Cccm nnn	I4/m	P42m	P4 ₂ /nbc		P6 ₂ /mcc	F43m	
	2 ₁ 2 ₁ 2 ₁	bc2 ₁		I4 ₁ /a	P4c2	P4 ₂ /nbc		P6 ₂ /mcc	I43m	
		ca2 ₁				P4 ₂ /nbc		P6 ₂ /mcc		